Multiobjective Optimization Using Adaptive Pareto Archived Evolution Strategy

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Abstract

This paper proposes a novel adaptive representation for evolutionary multiobjective optimization for solving a stock modeling problem. The standard Pareto Achieved Evolution Strategy (PAES) uses real or binary representation for encoding solutions. Adaptive Pareto Archived Evolution Strategy (APAES) uses dynamic alphabets for encoding solutions. APAES is applied for modeling two popular stock indices involving 4 objective functions. Further, two bench mark test functions for multiobjective optimization are also used to illustrate the performance of the algorithm. Empirical results demonstrate APAES performs well when compared to the standard PAES.

1 Introduction

Many multiobjective optimization techniques using evolutionary algorithms have been proposed in recent years. Pareto Archived Evolution Strategy (PAES) is one of the most important algorithms for multiobjective optimization. PAES is an simple evolutionary algorithm which can use real or binary representation of solutions. This paper proposes an adaptive representation of solutions for the standard PAES algorithm. This improvement has been inspired by Adaptive Representation Evolutionary Algorithm (AREA) proposed in [1]. Following this model, each PAES solution will consist of a pair (x, B), where x is a string encoding object variables and B specifies the alphabet used for encoding x. x is a string of symbols over an alphabet $\{0, \ldots, B\text{-}1\}$, $B \geq 2$. Mutation can modify object variables as well as the last position (fixing the representation alphabet). Some numerical experiments are performed to illustrate the APAES approach. In the first experiment APAES is used for modeling Nasdaq and Nifty stock indices [3], where the objective is to optimize four performance measures for predicting stock index values. Our previous research works ([3], [2]] involving Artificial Neural Networks (ANN), Neuro-Fuzzy (NF) model, Support Vector Machines (SVM), Difference Boosting Neural Network (DBNN) and Multi-Expression Programming (MEP) clearly illustrate that none of the considered techniques could find an optimal solution for all the four performance measures namely Root Mean Squared Error (RMSE), Correlation Coefficient (CC), Maximum Absolute Percentage Error (MAP) and Mean Absolute Percentage Error (MAPE). We combine the results obtained by these five techniques as an ensemble using APAES and PAES with the task of optimizing all the four objectives.

In the second experiment, we used two well known multiobjective optimization test functions (ZDT4 and ZDT6)

([5], [6]) to illustrate the algorithm performance.

The paper is structured as follows: Section 2 presents both PAES and APAES algorithms. In Section 3 numerical experiments with these two techniques are performed. Some conclusions are also provided towards the end.

2 PAES and Adaptive PAES

2.1 PAES algorithm

Knowles and Corne [4] have proposed a simple evolutionary algorithm called Pareto Archived Evolution Strategy (PAES). In PAES one parent generates by mutation one offspring. The offspring is compared with the parent. If the offspring dominates the parent, the offspring is accepted as the next parent and the iteration continues. If the parent dominates the offspring, the offspring is discarded and the new mutated solution (a new offspring) is generated. If the offspring and the parent do not dominate each other, a comparison set of previously nondominated individuals is used.

For maintaining population diversity along Pareto front, an archive of nondominated solutions is considered. A new generated offspring is compared with the archive to verify if it dominates any member of the archive. If yes, then the offspring enters the archive and is accepted as a new parent. The dominated solutions are eliminated from the archive. If the offspring does not dominate any member of the archive, both parent and offspring are checked for their nearness with the solution of the archive. If the offspring resides in the least crowded region in the parameter space among the members of the archive, it is accepted as a parent and a copy is added to the archive. The standard PAES algorithm is described as follows:

Standard PAES

repeat
  Generate initial random solution *c* and add it to archive
  Mutate *c* to produce *m* and evaluate *m*
  if *c* dominates *m*
    discard *m*
    else
    if *m* dominates *c*
      then replace *c* with *m* and add *m* to the archive
      else
      if *m* is dominated by any member of the archive
        discard *m*
        else apply test (*c*, *m*, archive) to determine which becomes the new current solution and whether to add *m* to the archive
      endif
    endif
  endif
until a termination criterion has been reached

2.2 Adaptive Pareto Archived Evolution Strategy (APAES)

APAES can be considered as an adaptive representation of the standard PAES. The following solution representation is used: each solution consists of a pair (x, B), where x is a string encoding object variables and B specifies the alphabet used for encoding x. x is a string of symbols over an alphabet $\{0,\ldots, B\text{-}1\}$, $B \geq 2$. The length of x is not fixed; for different value of the alphabet part different lengths for x are considered. In our experiments (see Section 3.4, Table 4) we used a precision representation so that the chromosome length is 30 when the alphabet is 2 (when we have binary representation). Of course, for greater values of B chromosome length is smaller.

And example of such a solution is C = (x = 30145413, B = 6) (This is an example only and it doesn't necessary means that the length of x is 8 if the alphabet is 6).

Mutation can modify object variables as well as the last position (fixing the representation alphabet).

When the changing gene belongs to the object variable sub-string (x – part of the chromosome), the mutated gene is a symbol randomly chosen from the same alphabet.

If the position giving B is changed, then the object variables will be represented using symbols over the new alphabet, corresponding to the mutated value of B. **Example**

Consider the chromosome C represented over the alphabet B = 8:

C = (x = 631751, B = 8).

If a mutation occurs on the last position (B) then the mutated chromosome is:

C_2 = (x_2 = 209897, B_2 = 10).

Remark

C and C_2 encode the same value over two different alphabets (B = 8, B_2 = 10).

The modification which occurs by using the above solution representation for PAES algorithm consists in what follows: when the current solution c dominates the mutated solution m for a consecutive fixed number of times it means that the representation of current solution has no potential for exploring the search space from the place where it belongs. Therefore the representation of the current solution must be changed in order to ensure a better exploration. In this respect the alphabet part of solution is changed into another random value. APAES is described as follows:

Adaptive Pareto Archived Evolution Strategy

repeat
    Generate initial random solution *c* and add it to archive
    k = 0
    Mutate *c* to produce *m* and evaluate *m*
    if *c* dominates *m*
    then
        k = *k* + 1;
        if *k* = Maximum number of harmful mutations
        then
            change the representation for the current solution (i.e. mutate the alphabet over which the current solution is represented);
            k = 0
        else
        if *m* dominates *c*
        then replace *c* with *m* and add *m* to the archive
        else
        if *m* is dominated by any member of the archive
        then discard *m*
        else apply test (*c*, *m*, archive) to determine which becomes the new current solution and whether to add m to the archive
        endif
    endif
endif
until a termination criterion has been reached

3 Experiment Setup and Results

3.1 Experiment 1

We analyze the behavior of five different techniques for modeling the Nasdaq-100 and NIFTY stock market indices so as to optimize the performance indices (different error measures and correlation coefficient) and to find an ensemble combination of these techniques in order to further optimize the performance. The five techniques used in the experiments are: an Artificial Neural Network (ANN) trained using the Levenberg-Marquardt algorithm, Support Vector Machine (SVM), Difference Boosting Neural Network (DBNN) , a Neuro-Nuzzy (NF) model and Multi-Expression Programming (MEP). Readers may please consult [2], [3] for technical details of the implementations and for information related to the algorithms used. In order to find an optimal combination of these paradigms, the task is to evolve five coefficients (one for each technique) so as to optimize the four performance measures namely Root Mean Squared Error (RMSE), Correlation Coefficient (CC), Maximum Absolute Percentage Error (MAP) and Mean Absolute Percentage Error (MAPE). For this purpose, the problem is formulated as a multiobjective optimization problem. Results obtained by the evolved ensemble are compared with the results obtained by the five techniques.

The goal is to optimize several error measures namely minimum values for RMSE, MAP,MAPE and a maximum value for CC.

$$RMSE = \sqrt{\sum_{i=1}^{N} |P_{actual,i} - P_{predicted,i}|},$$

$$CC = \frac{\sum\limits_{i=1}^{N} P_{predicted,i}}{\sum\limits_{i=1}^{N} P_{actual,i}},$$

$$MAP = \max\left(\frac{|P_{actual,\ i} - P_{predicted,\ i}|}{P_{predicted,\ i}} \times 100\right),$$

$$MAPE = \frac{1}{N}\sum_{i=1}^{N}\left[\frac{|P_{actual,\ i} - P_{predicted,\ i}|}{P_{actual,\ i}}\right] \times 100,$$

where $P_{actual,i}$ is the actual index value on day i, $P_{predicted,i}$ is the forecast value of the index on that day and N = total number of days. The objective is to carefully construct the different intelligent paradigms to achieve the best generalization performance. Test data is then passed through these individual models and the corresponding outputs are recorded. Suppose the daily index value predicted by DBNN, SVM, NF, ANN and MEP are a_n, b_n, c_n, d_n and e_n respectively and the corresponding desired value is x_n. The task is to combine a_n, b_n, c_n, d_n and e_n so as to get the best output value that maximizes the CC and minimizes the RMSE, MAP and MAPE values.

3.2 Ensemble Approach

The proposed method is to evolve a set of five coefficients (one for each technique) in order to obtain a linear combination between these techniques so as to optimize the values of RMSE, CC, MAP and MAPE. We consider this problem as a multiobjective optimization problem in which we want to find solution of this form: ($coef_1$, $coef_2$, $coef_3$, $coef_4$, $coef_5$), where $coef_1$,..., $coef_5$ are real numbers between -1 and 1, so as the resulting combination:

$$coef_1 * a_n + coef_2 * b_n + coef_3 * c_n + coef_4 * d_n + coef_5 * e_n$$

would be close to the desired value (let us note this x_n). This problem is equivalent to finding the Pareto solutions of a multiobjective optimization problem (objectives being RMSE, CC, MAP and MAPE). We compare here results obtained by PAES and APAES for constructing ensembles.

3.3 Parameter Settings

The main parameters used in the experiments by PAES and Adaptive PAES are presented in Table 1.

Table 1. Parameters used by PAES and Adaptive PAES

Parameter	Value
Archive size	250
Number of function evaluations	125,000
Chromosome length (for binary representation)	30

Adaptive PAES uses 32 alphabets and precision representation is 0.000000001. Number of unsuccessful mutation after which the alphabet part of the chromosome is changed is 500. Empirical results obtained by all 5 paradigms and by ensembles using PAES and Adaptive PAES are presented in Table 2.

Results are graphically illustrated in Figures 1 and 2 for Nasdaq and Nifty stock indices respectively. Greater values for CC and lower values for RMSE, MAP and MAPE indicate a better convergence. For Nasdaq, APAES provided best values for RMSE and CC values but performed relatively well for MAP and MAPE. For Nifty stock index, APEAS performed well especially for MAP while giving comparative performance for RMSE and CC with respect to CC. For MAPE values, PAES performed slightly better than APAES.

3.4 Experiment 2

Test functions ZDT4 and ZDT6 proposed in [5][6] are used for experiments. The test function T4 contains 21^9 local Pareto optimal fronts. The test function T6 includes two difficulties caused by the non-uniformity of the search space: first, the Pareto optimal solutions are non-uniformly distributed along the global Pareto optimal front and second, the density of the solutions is lowest near the Pareto optimal front and highest away from the front. Parameters used for the experiments are presented in Tables 3 and Table 4 respectively.

We compare the results obtained by PAES and APAES using two performance measures: C and S metrics proposed in [6]. The results obtained by applying C metric are

Table 3. Parameters used by PAES

Parameter	Value
Chromosome length	900
Number of iterations	25,000
Archive size	100
Mutation probability	0.002

Table 4. Parameters used by APAES

Parameter	Value
Representation precision	0.000000001
Number of iterations	25,000
Archive size	100
Mutation probability	0.002
Number of unsuccessful mutations after which the alphabet is changed	100

presented in Table 5 and S metric results are graphically illustrated in Figure 3. Figure 3(a) and 3 (b) corresponds to test functions ZDT4 and ZDT6 respectively. As evident for both test functions, APEAS performed well when compared to PAES.

Table 5. Results obtained by applying C metric

	PAES	Adaptive PAES
Test function ZDT4		
PAES		1
Adaptive PAES	0	
Test function ZDT6		
PAES		0,0930233
Adaptive PAES	0,0707071	

The value C(A, B) = 1 applied for two set of solutions A and B means that all decision vectors in B are dominated by A. The opposite, C(A, B) = 0, represents the situation when none of the points in B are dominated by A. Greater values for C metric indicate first set is better than the second set.

4 Conclusions

A novel adaptive representation (APAES) for Evolutionary Multiobjective Optimization (EMO) by modifying the standard Pareto Achieved Evolution Strategy (PAES) is presented in this paper. APAES uses dynamic alphabets for encoding solutions when compared to the fixed binary representation in PAES. APAES is applied for modeling Nasdaq

Table 2. Performance comparison of the results obtained by PAES and APAES)

	SVM	NF	ANN	DBNN	MEP	APAES	PAES
Test results - NASDAQ							
RMSE	0.0180	0.0183	0.0284	0.0286	0.021	0.0160	0.01614
CC	0.9977	0.9976	0.9955	0.9940	0.999	0.999	0.998
MAP	481.50	520.84	481.71	116.98	96.39	95.58	94.976
MAPE	7.170	7.615	9.032	9.429	14.33	11.21	10.542
TEST results – NIFTY							
RMSE	0.0149	0.0127	0.0122	0.0225	0.0163	0.013	0.013
CC	0.9968	0.9967	0.9968	0.9890	0.997	0.999	0.999
MAP	72.53	40.37	73.94	37.99	31.7	25.55	29.75
MAPE	4.416	3.320	3.353	5.086	3.72	3.01	2.910

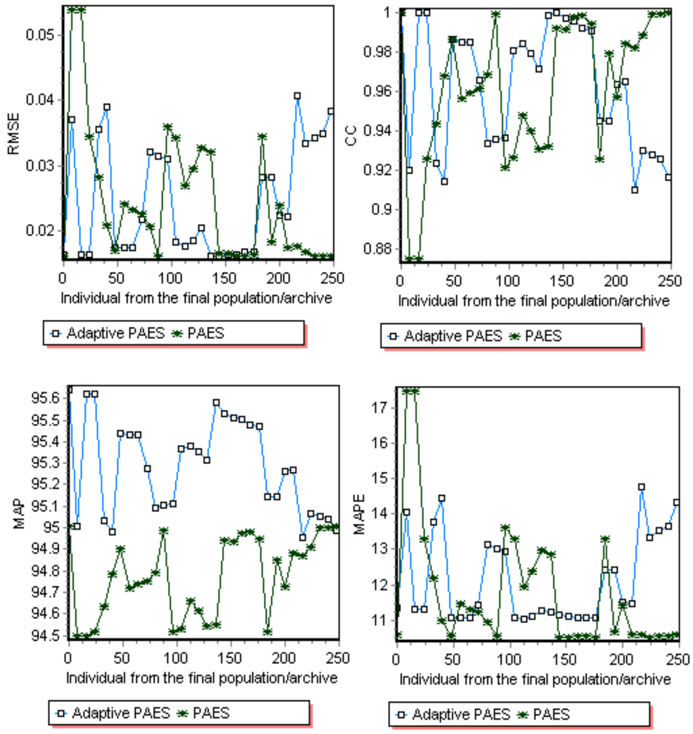

Figure 1. Values obtained by APAES and PAES for Nasdaq test data

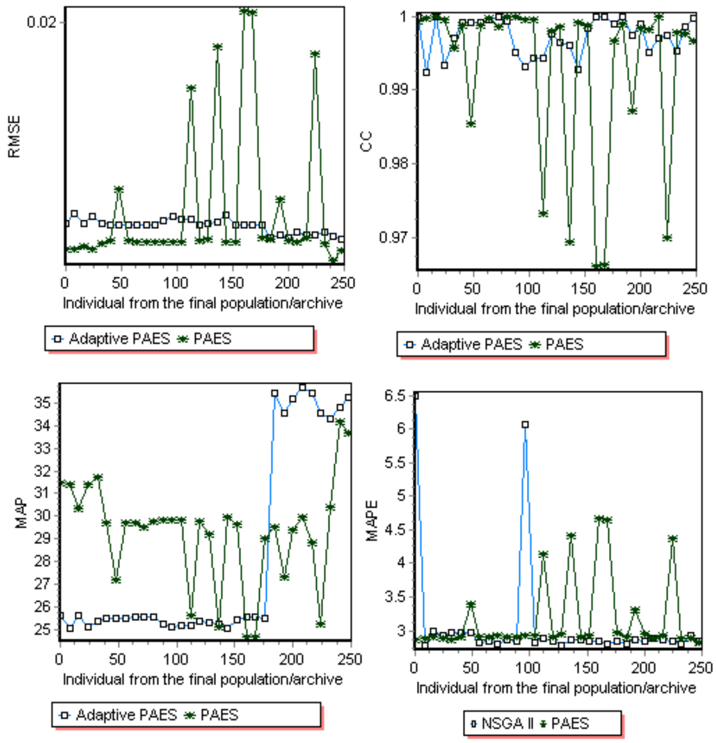

Figure 2. Values obtained by APAES and PAES for Nifty test data

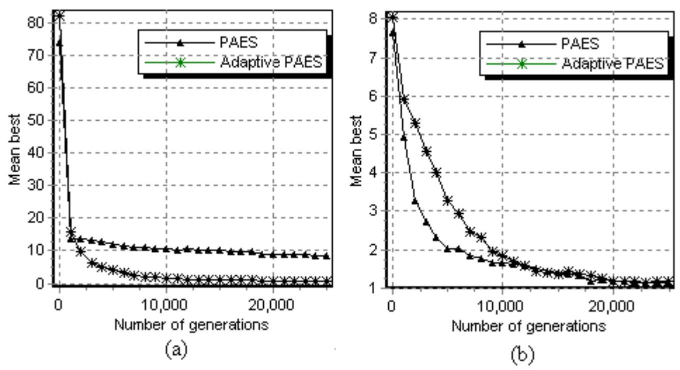

Figure 3. Results obtained by PAES and APAES for test functions ZDT4 and ZDT6 by applying S metric

and Nifty stock indices and the empirical results clearly indicates that the proposed EMO method is very promising. Experiments using the two bench mark test functions also reveal that APAES could give better solutions faster than PAES (faster convergence). Our future research is targeted to guide the APAES search procedure (for faster convergence) using some hybrid meta-heuristic approaches.

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