Pareto-Morphology for Color Image Processing: A Comparative Study of Multivariate Morphologies[1, 2]

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Abstract—This paper presents an approach to the generalization of grayscale morphology to color images. Attaining such a generalization is strongly related to the issues of multivariate ordering and to the Pareto sets of multiobjective optimization. Some ranking schemes for multivariate data are recalled. For color morphology, the most important underlying ranking scheme is reduced ordering (also referred to as total ordering). Also, there is the partial ordering, which gives the important class of Pareto-Morphologies. Since partial ordering by Pareto sets commutes with reduced ordering, a so-called Pareto-Morphology is defined as a generalized multivariate morphology, for which the results will not change, if its computations are restricted to the Pareto set of the (local) neighborhood of a pixel. By further applying the concept of fuzzy subsethood to color values, a Pareto-Morphology can be designed, which is not based on reduced ordering, hence, it provides a manner for native color treatment. The properties of this newly-proposed Fuzzy-Pareto-Morphology and examples of its application for the processing of color textile images are given.

1. INTRODUCTION

Mathematical morphology can be considered as a theoretical and practical means for analyzing spatial structures. It comprised a versatile toolset of techniques for image processing, whose usefulness has been proven for the processing of binary images and grayscale images as well. Operations of mathematical morphology are image-to-image transformations based on a structuring element, which acts like a probe sensitive for structural information. As a result of the operation, some image features might be enhanced, suppressed, or preserved [12].

Basically, there are two morphological operations, dilation and erosion, which are used for the definition of more complex morphological operations. Nowadays, definitions of dilation and erosion are fixed for the treatment of binary images and grayscale images. Other concepts include the generalization of these basic definitions or fuzzy logic (consider, e.g., [2, 7, 11]).

However, requirements for a generalized dilation as an image-to-image operation, which employs a structuring elements, are still under discussion. As suggested in [10], there should be three key ideas, based on which the dilation is defined:[3] an idea of ranking due to a sort order; in idea of a supremum due to this ranking; and the possibility of admitting an infinity of operands.

Color image processing is of essential importance in order to increase robustness, versatility, and reliability of technical vision systems. As exemplified by human perception abilities, color is more than a simple "add-on" to grayscale images.

Two questions are considered to be of basic importance for color image processing, the question of color representation, which is strongly relate to color spaces, and the question of appropriate color image processing operations. While investigations on the first question resulted in a wide variability of technical, psychological, or theoretical important color models (e.g., HSI, RGB, CVYK, Lab, to name but a few…), research on the second question has been performed in a more restricted manner. Some basic problems related to color image processing are: multivariate nature of color data, which complicates the extension of some grayscale operations to color images (e.g., convolution, mathematical morphology); and the dual nature of human (or mammalian) color perception sensitiveness: being highly sensitive to smallest "color artifacts", and being highly insensitive for luminescence variations within images (e.g., under varying lightning conditions) at the same time.

This complicates and restricts possible definitions for versatile operations on color images. Neither Laplacian, Sobel nor thresholding found commonly accepted, suitable counterparts in color spaces so far. This holds for mathematical morphology as well.

This paper deals with the definition of dilation (and erosion) within the context of color image processing. The fundamental lack of a "natural sort order" of multivariate data and the numerical differences due to the choice of different color spaces make it hard or even impossible to define something like a "color morphology". But it could be expected to transfer a large number of grayscale morphological techniques to color

[1] This paper was submitted by the authors in English.
[2] The original color figures provided by the authors are presented here in black and white. We apologize for this inconvinience.
[3] Generally, when the definition of a dilation is fixed, the erosion is defined as the complementary operation.

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images. It could be expected to design highly color-specific operations from morphology as well.

Very few of past works dealt with such extensions. In [5], the issue is intensively discussed and a definition of morphology is presented, which will be generalized in this paper to a larger class of color morphologies, each of which is based on its unique definition of the dilation operation. In [13] and [4], the approach presented in [5] is generalized and based on a different formal viewpoint. In [3], a color morphology for the processing of label images (i.e., images, wherein each pixel position is labeled by a color, indicating, e.g., class membership) is presented. This approach is very useful for the morphological treatment of pseudo-colorized segmentation results, but it lacks some consistency within its basic definitions and it assumes that only a few colors within the image are present.

The core of this paper presents a new dilation for multivariate data (including color images), which is merely not based on reduced ordering, thus, it comprises a "native" treatment of color images. It is based on the concept of fuzzy subsethood, as introduced in [9] and interpreted as a fuzzy dominance relation. The fuzzy data of mutual degrees of dominance are fused by a minmax operation.

This paper is organized as follows. Section 2 considers various approaches to a multivariate or color morphology and some requirements for generalized dilation operations as a whole (Section 2.1). Based on Barnett's classification of multuvariate ranking schemes [1], all approaches can be subdivided into three classes (Section 2.2). Sections 2.2.1 and 2.2.2 deal with generalized morphologies based on marginal and reduced ordering, which cover most of the multivariate morphologies proposed so far. As a new aspect, given in Section 2.2.3, partial ordering is reflected by the concept of a Pareto-Morphology. In addition, in Section 2.2.5, the complete ordering scheme [13] is considered, which does not fit into one of the categories of Barnett. Then, Section 2.3 considers the proposed Fuzzy-Pareto-Morphology (FPM). At first, in Section 2.3.1, the geometric sets-as-points approach of Kosko is recalled. The fuzzy dominance relation is introduced in Section 2.3.2. In Section 2.3.3, the definitions of FPM are given along with an example, Section 2.3.4 gives some properties of the FPM together with their proofs, and Section 2.3.5 introduces some other color image processing operations based on the same concept. Then, in Section 3, the approaches are discussed. The paper ends with a short summary in Section 4.

2. APPROACHES TO MULTIVARIATE MORPHOLOGY

2.1. Requirements for a Generalized Dilation

The basic operations of mathematical morphology, *dilation* and *erosion*, are defined in terms of the complete lattice theory [11]. A set $\mathcal{L}$ with a partial ordering is a complete lattice if each of its subsets possesses a supremum max and an infimum min according to the partial ordering. Given two complete lattices $\mathcal{L}$ and $\mathcal{M}$, a mapping $f: \mathcal{L} \longrightarrow \mathcal{M}$ is a *dilation*, if it commutes with the supremum operator, i.e., for $S_i \in \mathcal{L}$ $f(\max_L[S_i]) = \max_M[f(S_i)]$, whereas $\max_L$ and $\max_M$ refer to the definition of the supremum operator on the complete lattices $\mathcal{L}$ and $\mathcal{M}$, respectively. It is called an *erosion*, if it commutes with the infimum operator.

If $\mathcal{M}$ and $\mathcal{L}$ are identical, the supremum operator itself is a dilation, and, conversely, each dilation gives a partial ordering of $\mathcal{L}$, for which it is a dilation as well. In this case, the definition of a dilation (and an erosion, too) can be considered to be given by specification of a ranking scheme.

Color images can be considered as given by a color image function $p: \mathcal{D}p \subset \mathbf{Z}^2 \longrightarrow \{0, \ldots, p_{\max}\}^3$, where $\mathcal{D}_p$ is the image domain, usually a rectangular subset of the coordinate plain, and $p_{\max}$ is the maximum intensity value for one color channel. The lattice $\mathcal{L}$ is given by the set of all possible triples of color value components, i.e., a given instance of a color space. Usually, the second lattice $\mathcal{M}$ is given by the same color space. The chosen color space should be fixed in the following, and all of its channels are assumed to have the same resolution.

In order to define a dilation (or erosion) operation, a concept for a supremum (or infimum) is necessary. In case of color data, this is implied by a multivariate ranking scheme. If a set operator $\mathbf{P}$, which assigns a supremum to a set of color values (what will be discussed in the next subsection) is assumed to be given, the definition goes on as follows. Let a and b be structuring elements, e.g., defined as a set of offsets with a, $b \subset \mathbf{Z}^2$ and with respect to a central point. Thus, a structuring element defines a neighborhood M for each pixel of the image. Then, a *color dilation* $\oplus$ is a set function $\oplus$: $M \subset \mathcal{L} \longrightarrow m \in \mathcal{L}$, which commutes with the supremum operator $\mathbf{P}$. Similarly, a color erosion $\ominus$ is a set function which commutes with the infimum operation. If both operations are dual operations (i.e., $\ominus = \mathbf{C} \oplus \mathbf{C}$ with $\mathbf{C}$ being an image complement operation), color dilation and color erosion together give the basic operations of a color morphology. In the following, only the case of color dilation is studied. The discussions may be equally applied to the case of a color erosion.

The color dilation $\oplus$ will assign a new color value $p_{\text{new}} \in \mathcal{L}$ at each image position (x, y), which is computed from the color values of the pixels in the neighborhood of (x, y). More formally,

$$p_{\text{new}}(x, y) = \oplus \circ \{p(k, l) | k = x + i, l = y + j, i, j \in a\}.$$

Not all properties of a generalized dilation follow from the requirement, that a dilation commutes with the supremum operator. For judging the practical importance of a generalized dilation, especially in the context of color image processing, the following properties will be checked for several approaches to multivariate morphology:

1. The color dilation should be *color-proof*, i.e., **P** should just select one color value out of the set M of color values within the neighborhood of a pixel. This is quite important for the processing of color images, due to the fact, that newly introduced colors within the result image might appear as artifacts of cluttering. This requirement could be relaxed by using the HSI color model and preventing introducing new H components only. In [13], this property is referred to as *vector-preserving*.

2. A color dilation should be an extensive operation, i.e.,

$$p \oplus a \geq p,$$

where the meaning of ≥ is used according to the key idea of sorting, as it was mentioned in the introduction.

3. When $\oplus_B$ assigns standard binary dilation, the color dilation should be *compatible* with the operands of this operation, i.e.,

$$(p \oplus a) \oplus b = p \oplus (a \oplus_B b),$$

where a and b have the same central point. Some algorithms of mathematical morphology make use of this property for faster computation of the dilation by decomposing (if possible) the structuring element into simpler ones.

4. In the context of multivariate data, a color dilation should become a standard grayscale dilation, if the difinition of ⊕ is restricted to the one-dimensional (or univariate) case.

There are much more properties of a generalized dilation or erosion to consider (see [2] for a comprehensive collection). However, generalizing the concept of standard mathematical morphology to another application field (such as color images) is an inductive process, not a deduction. Once the basic requirement of a dilation (to commute with the supremum) is fulfilled, all other properties will be either strongly related to this property, directly follow from it, or they will provide a means to estimate possible application fields of the newly designed operations. They will never give the proof that the given definition of a generalized morphology is wrong.

2.2. Multivariate Ordering

For a lattice $\mathcal{L}$ to be complete, an operation must be given which assigns a supremum to each subset of the lattice. While there is a "natural" sort order for univariate (or one-dimensional vector) data, the extension of ordering to multivariate data is a great controversy. In [1], multivariate ranking schemes known so far were classified into four categories: marginal ordering, reduced ordering, partial ordering, and conditional ordering. Since the operation, which assigns the supremum to a set by means of a given ranking scheme in the lattice $\mathcal{L}$ can be considered as a multivariate dilation from the lattice $\mathcal{L}$ in $\mathcal{L}$, these categories can be used for classifying multivariate morphologies as well. Only conditional ordering seems to be inappropriate for designing new morphologies, because its ordering scheme is based on statistical properties of the vectors, which are to be sorted. But, in practive, most structuring elements are not big enough for revealing the underlying statistical distribution. We leave this question open for further research.

In the following, these ranking schemes will be given in more detail, and their relations to the accompanying dilations will be discussed. For illustration purposes, the multivariate example set

$$F = \{(2, 3, 5), (4, 1, 6), (2, 2, 4)(1, 1, 1)\}$$

will be used.

2.2.1. Marginal ordering. An example for a color dilation based on marginal ordering (referred to as a marginal morphology) is given by applying a univariate supremum function **P** component-by-compontent:

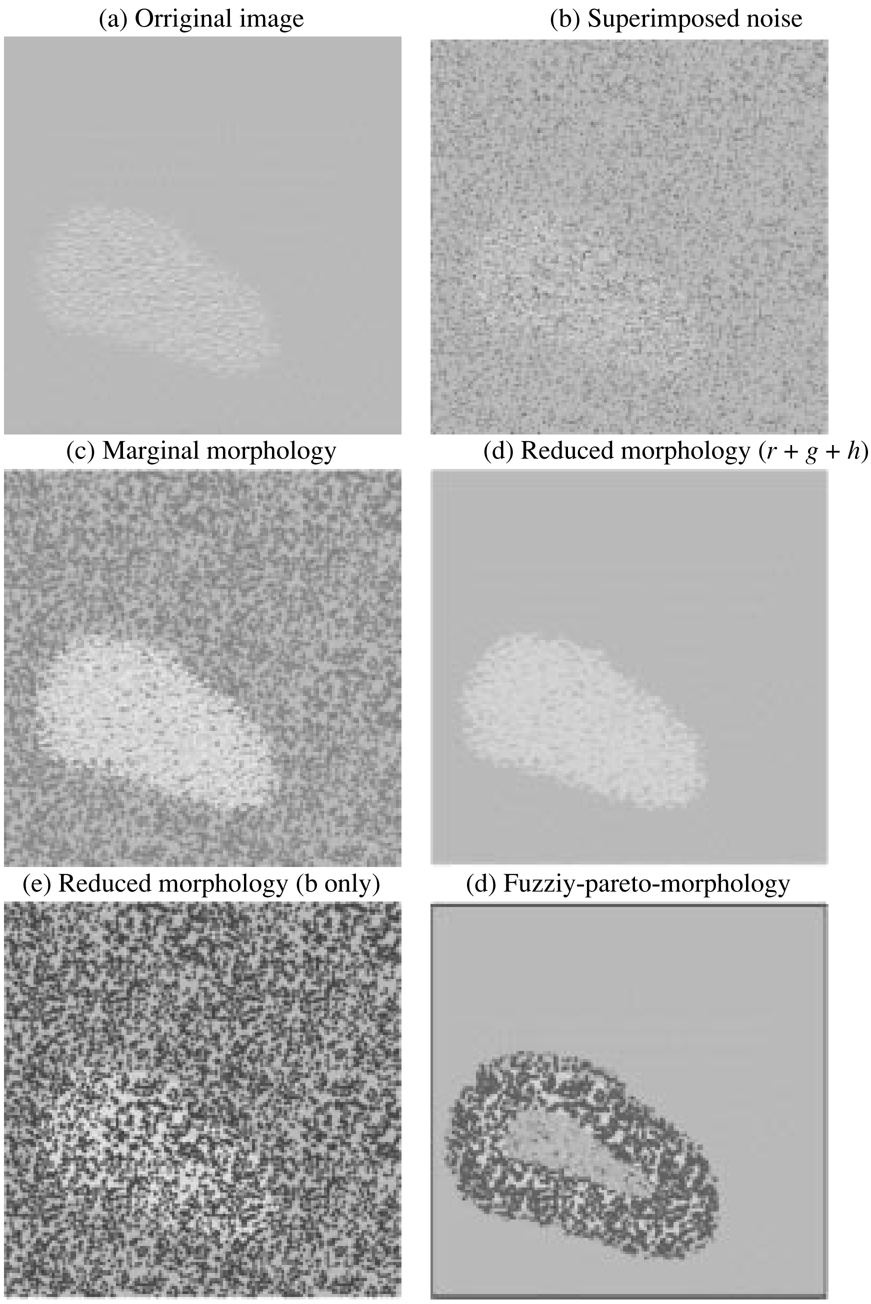
$$\mathbf{P}_{\text{marg}} \circ \{(p_x, p_y, p_z)\}$$
$$= (\mathbf{P} \circ \{p_x\}, \mathbf{P} \circ \{p_y\}, \mathbf{P} \circ \{p_z\}).$$

For the set F, this procedure gives the result vector

$$(\max[2, 4, 2, 1], \max[31, 2, 1], \max[5, 6, 4, 1])$$
$$= (4, 3, 6),$$

which in not an element of the example set. Marginal ordering, if used as color dilation, will produce new color values within the result image, thus violating property 1. Colorplate 1c shows the marginal dilation of the image of colorplate 1a with superimposed noise (colorplate 1b). The disturbed color appearance, as a result of newly appearing color values, can be clearly seen.

In general, a marginal ordering assumes the color image to be split up into a set of channels C_i (e.g., by using the vector index operator, which assigns to vector $\vec{v}$ its ith component), in a manner from which the color image can be fully recovered by merging the channels. Then, a univariate ordering is applied to each channel, and the processed channels are merged to the result image. Well-known examples for such splittings are the RGB-, HSI-, Lab-, and CMYK-decomposition. An extreme case of channel splitting was presented in [3]. There, to each color, which is present in the color image, its own channel is assigned, i.e., the number of channels is equal to the number of different colors in the image. The numeric value of the channel of color value c at position (x, y) is 1, if the color image has color value c at this position, 0 otherwise. The color morphology derived from this channel splitting was



Colorplate 1. Effects of different color morphologies on synthetic image.

applied to the postprocessing of color-labeled segmentation images. It allows for using *a priori* knowledge of segment properties to be employed for the composition of morphological operators. A new class of morphological operators is presented in [3], including the so-called *tunneling* and *bridging* operations; this definitions can only be given for marginal morphologies.

From this, the objections against marginal morphologies, as given, e.g., in [5] and [13], should be weighted more carefully in future works.

Marginal morphologies do not fulfill property 1, but they fulfill properties 3 and 4. Due to its own ordering scheme, they also fulfill property 2 (see Section 2.1).

2.2.2. Reduced ordering. By reduced ordering, a scalar parameter function f is computed for each color value of the multivariate data set S. The ordering is performed according to the resulting scalar values,

$$\mathbf{P}_{\mathrm{red}} \circ \{(p_x, p_y, p_z)\} = \underset{p \in M}{\operatorname{argmin}}[f(p)],$$

Either function f could be used, e.g., $f(x, y, z) = x + y + z$, $f(x, y, z) = \max[x, y, z]$ or $f(x, y, z) = xy - z$. Color-

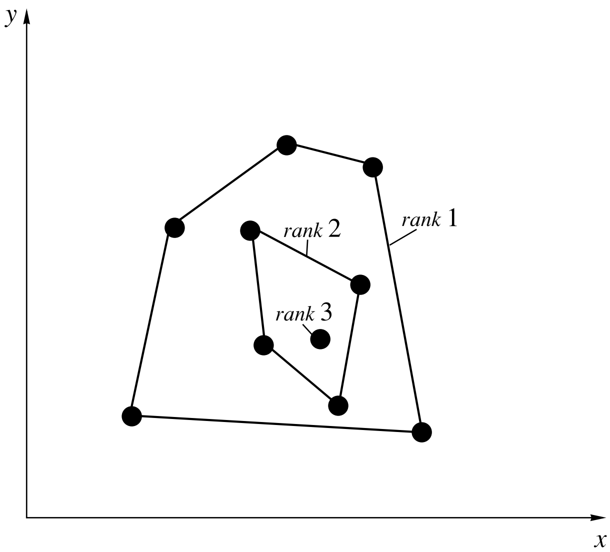


Fig. 1. Partial ordering of 10 points by convex hul operator.

plates 1d and 1e give two examples, using the parameter functions $f(x, y, z) = x + y + z$ as an example for a symmetric function f, and $f(x, y, z) = z$, which is not symmetric. In these examples, x, y, and z are the numerical values of the red, green, and blue components of the color value, respectively.

For the exemplary set, if $f(x, y, z) = z + y + z$, the reduced dilation gives the vector argmax[$f(2, 3, 5)$, $f(4, 1, 6)$, $f(2, 2, 4)$, $f(1, 1, 1)$] = argmax[2 + 3 + 5, 4 + 1 + 6, 2 + 2 + 4, 1 + 1 + 1] = argmax[10, 11, 8, 3] = (4, 1, 6). Equally, if $f(x, y, z) = x \cdot y$, this procedure selects the example vector with the largest product of x and y component, which is the first vector (2, 3, 5).

Total ordering [4] can be considered as a reduced ordering. If the sort order of a set of vectors $\vec{v}_i$ is explicitly given as $\vec{v}_{(1)}, \ldots, \vec{v}_{(n)}$ (or by means of a space-filling curve, as it was proposed in [4]), the parameter function is just $f(\vec{v}_{(i)}) = n - i$. However, not each of reduced ordering gives a total ordering, due to the possibility, that several values of vector components give the same value for f. The problem can be patched by sub-ordering vectors with the same f-value due to another criterion, hence, each partial order implied by a reduced ordering can be mapped onto a set of total orderings. From the authors' point of view, this is a rather technical task. If, e.g., the parameter function is $f(x, y, z) = xyz$, then the vectors (2, 4, 2) and (1, 1, 16) will give the same value 16 for f. A total ordering would assign different positions to both vectors in the ordered sequence of vectors, hence, it avoids ambiguities. What is needed for a reduced ordering is an additional criterion for deciding, which of both vectors comes first. This can be simply given (e.g., by adding the sum of the components to f giving $f(x, y, z) = xyz + x + y + z$). If further ambiguities occur, f must be further modified. There will always be a modification of f, which leads to a total ordering. We could call the modified final version f_{tot} of f "totalization" of f. Totalization is not important for practical reasons (result images will not look very different), but it complicates mathematical proofs and the justification of properties. Total ordering gives a much simpler framework.

An essential property of reduced morphologies (and total ordering based morphologies as well) is that the "strength" of each color value in the "competition" among color values, which takes place at each computational step of a dilation or erosion (the competition is won by the supremum or infimum) is assigned beforehand. It does not depend on the color context. There is no reasonable objection against this behaviour of reduced morphologies, but it constraints the possible outcomes of a morphological processing chain.

Also, it is not expected for a reduced morphology to perform better than a grayscale morphology applied to the mapping of the color image onto a single grayscaled channel by f (including an appropriate normalization). The channel represents the idea of intensity, which underlies the chosen f. Each result, derived from the processed color image, could be derived from the processed grayscale channel as well. This explains the qualification of such ordering schemes by [1] as *reduced*. Essential color information could get lost during the processing, without gaining anything with respect to the processing goal, thus, violating Marr's principle of Graceful Degradation.[4]

Reduced morphologies fulfill properties 1, 3, and, according to the definition of f, also property 4 (see Section 2.1). Their relation to the property 2 depends on the concept of a supremum. If this is the maximum of n values, property 2 is fulfilled.

2.2.3. Partial ordering. The third ranking scheme, partial ordering,[5] differs from marginal and reduced ordering by possibly selecting more than one value out of the set of the unsorted values. Partial ranking makes use of a maximum set operator $\mathbf{P}$: $M \longrightarrow 2^M$, which assigns an element of the powerset of the set M to M. A typical choice for a maximum set function $\mathcal{P}$ is the convex hull of the data set. Then, a sequence of subsets, M_i of M is derived according to the following rules:

1. $M_1 = \mathbf{P} \circ M$;
2. $R_1 = M - M_1$;
3. $M_{k+1} = P \circ (R_k - M_k)$;
4. $R_{k+1} = R_k - M_k$;

which are applied until $R_l = \varnothing$. All elements belonging to M_i are said to be of rank i by this partial ordering. Elements with the same rank i cannot be further distinguished by such a method. Partial ordering iteratively strips off maximum subsets of the data set, until the remaining set is empty (see Fig. 1).

[4] Degraded input should deliver at least some of the answers.

[5] Here, partial ordering has a different meaning than the partial ordering of a lattice $\mathcal{L}$.

In order to use partial ordering to define a multivariate dilation, an additional procedure has to be supplied for selecting exactly one value out of the set of the stripped-off maximum subsets. This has been considered as an important disadvantage of the partial ordering [5].

2.2.4. The Pareto ordering theorem. Pareto ordering, which will be described next, gives another instance of a partial multivariate ordering scheme. It has an important relation to reduced ordering.

For two vectors $\vec{a}$ and $\vec{b}$, it is said that $\vec{a}$ *dominates* $\vec{b}$, when each component of $\vec{a}$ is at least as large as the corresponding component of $\vec{b}$, and at least one component is larger

$$\vec{a} >_D \vec{b} \longleftrightarrow \forall i(a_i \geq b_i) \wedge \exists k(a_k > b_k). \quad (1)$$

The subset of all vectors of the set M, which are not dominated by any other vector of M, is the Pareto set (also Pareto front). Let $\mathbf{P}_p$ be the subset operator, which assigns the Pareto set to a set of vectors. The Pareto operator for univariate data is the maximum operation.

In the example set, the vector (2, 3, 5) dominates the vectors (2, 2, 4) and (1, 1, 1), but it is not dominated by another vector of the example set. Hence, (2, 3, 5) as well as (4, 1, 6) constitute the Pareto front of the example set, or the set of vectors of rank 1 (M_1). Then, R_1 is the set {(2, 2, 4), (1, 1, 1)}. Here, (2, 2, 4) dominates (1, 1, 1). The set M_2 of vectors of rank 2 is {(2, 2, 4)}, and the set of vectors of rank 3 is {(1, 1, 1)}.

The Pareto set describes the possible solutions of a multiobjective optimization problem [6]. According to the problem statement, every solution, which gives an element of the Pareto set, when their multiple criteria are computed, will be optimal in this generalized sense.

The following theorem holds for a partial ordering based on the Pareto operator as maximum set function:

Theorem 1. *A reduced ordering with a monotonic increasing scalar function commutes with the Pareto ordering.*

Proof. The theorem states, that the vector $\vec{v}_{\max}$ with the maximum value $f_{\max}$ of f is an element of the Pareto set. If not, there is a vector $\vec{v}_d$ which dominates $\vec{v}_{\max}$. Since $f(\vec{v}_{\max})$ is the maximum value of f, $f(\vec{v}_d) < f_{\max}$. From dominance relation, it follows that all components of $\vec{v}_d$ are not smaller than the corresponding components of $f(\vec{v}_{\max})$ and at least one component is larger. But, by monocity of $f(\vec{v}_d) < f_{\max}$, it follows that $f(\vec{v}_d) \geq f(\vec{v}_{\max})$, which is a contradiction.

If f is not monotonic, the theorem does not hold. Consider, for example, the function $f(x, y) = \sin\frac{\pi}{2}x + \sin\frac{\pi}{2}y$ and its values $f(1, 1) = 2$, $f(2, 2) = 0$, and $f(1, 3) = 0$. Since the Pareto set is given with {(2, 2), (1, 3)}, the selected point with the maximum value of f is (1, 1).

Hence, Pareto ordering gives a partial ordering on the set lattice $\mathcal{L}$ of n-dimensional vectors. Each dilation, which commutes with the Pareto ordering, is defined as a *Pareto-Morphology.* The set of all Pareto-Morphologies is not empty, since monotonic reduced morphologies belong to this class. The question now is, whether it also holds, that every dilation, which commutes with Pareto ordering, has to be a reduced morphology.

2.2.5. Complete ordering. In [13] an ordering scheme is presented, which does not directly fit into one of the categories of Barnett. A supremum operator $\mathbf{P}$ gives a complete ordering, if it fulfills the following conditions:

1. If S is a subset of the lattice $\mathcal{L}$, then there is an $x_i \in S$ with $\mathbf{B}(S) = x_i$.

2. For all x_1 and x_2 from $\mathcal{L}$ $\mathbf{P}\{x_1, x_2\} = x_1$ and $\mathbf{P}\{x_2, x_1\} = x_2$ iff $x_1 = x_2$.

3. For all $x_1, x_2, x_3 \in \mathcal{L}$ from $\mathbf{P}\{x_1, x_2, x_3\} = x_1$ and $\mathbf{P}\{x_2, x_3\} = x_2$ it follows that $\mathbf{P}\{x_1, x_3\} = x_1$.

Each total ordering scheme fulfills these properties, as well as the totalization of a reduced ordering. However, not each of supremum operators with these properties must give a total ordering, because none of these properties links the cases of different numbers of elements, to which the supremum operation is applied. For example, the relation, that $\mathbf{P}\{x_1, x_2\} = x_1$ follows from $\mathbf{P}\{x_1, x_2, x_3\} = x_1$, which is obviously true for the univariate case, cannot be derived from these three properties, since none of them is concerned with terms like $\mathbf{P}\{x_1, x_2, x_3\}$. Despite the fact, that each set of vectors could be ordered pairwise, the ordering for three or more values at once may depend on all values which are to be sorted. This is a feature, which is strongly related to the fact, that the data are multivariate and not univariate. For multivariate data, it could be possible that the supremum of, e.g., (x_1, x_2, x_3) is x_1 but the supremum of (x_1, x_4, x_3) is x_3.

Since the supremum of a compete ordering is not related to the magnitudes of the components of the vectors, it can not be decided from these properties, whether the dilation based on this supremum operator (called δ-operator in [13]) gives a Pareto morphology or not.

Complete orderings include the set of all total orderings, but there may be other complete ordering schemes, which does not give a total ordering. The same question as for Pareto morphologies appears.

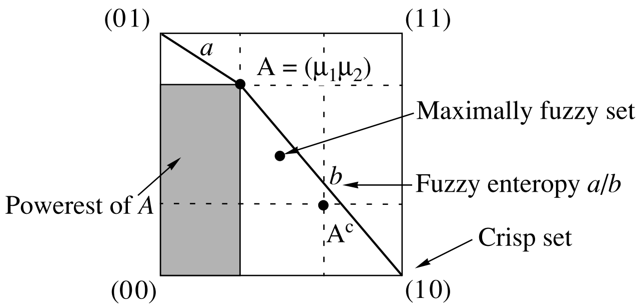


Fig. 2. Geometric representation of fuzzy sets as points in the unit square.

Does it follow from the fact, that an ordering is complete, that it must be a total ordering as well?

The answer to both questions is negative. In fact, there is a dilation, which gives a Pareto morphology, and is not based on reduced ordering; and which uses a complete ordering, which does not give a total ordering as well. This dilation, based on fuzzy concepts for ranking values, will be presented in the next section.

2.3. Fuzzy-Pareto-Morphology

2.3.1. Sets-as-points approach to fuzzy sets. Generally, a fuzzy set is given by the membership degrees of its elements

$$\mathcal{M} = (\mu_1, \mu_2, \ldots, \mu_n).$$

Kosko noted [9] that this gives a functional description of a fuzzy set. He urged for a visualization of fuzzy sets, which may help to understand them better. The idea is to represent fuzzy sets as points in the n-dimensional unit square (or unit cube) by using the membership degrees as coordinates. Some definitions can be given geometrically, as shown in Fig. 2 for the case of two-dimensional fuzzy sets. If the fuzzy set is $A = (\mu_1, \mu_2)$, then A^C is the complementary fuzzy set. The corners of the square represent the "crisp" sets, with membership values out of the set $\{0, 1\}$. The ratio of the length a of the shortest connection of A to a corner to the length b of the longest connection to a corner is defined as *fuzzy entropy.* The fuzzy entropy is lowest (0) for crisp sets and maximum for the midpoint of the square.

The rectangle, which is spanned by the empty set (0, 0) and the fuzzy set A (the dark area in Fig. 2), is the powerset of A, since it contains all fuzzy sets $B = (\nu_1, \nu_2)$, for which $\nu_1 \le \mu_1$ and $\nu_2 \le \mu_2$. By membership domination, these are the fuzzy subsets of A [14]. Its area will be denoted by $M(A)$.

Starting off from the relation for ordinary sets

$$A \subset B \text{ iff } A \in 2^B$$

in [9], a fuzzy extension of the term subsethood is derived, giving the degree of a fuzzy set A to which it is a subset of a fuzzy set T. This measure is the amount of fuzzy subsets of A, which are also fuzzy subsets of T, to the total count of fuzzy subsets of A:

$$S(A, T) = \frac{M(A \cap T)}{M(A)}.$$

2.3.2. Fuzzy dominance relation. Color values can be considered as fuzzy sets, with each of the numerical values of a component being the degree of membership to this basic component color, for at least three reasons:

—Color descriptions are not strictly defined. Names like "pale blue" or "velvet blue" can be interpreted as fuzzy descriptions as well.

—Fuzziness of colors is related to the manner in which colors are perceived. Color response curves can be modelled by Gaussian-like shaped receptive fields, proving that the perception of a certain color is not restricted to a certain wavelength, but is activated by nearby wavelengths as well. In [8], it was shown that fuzzy logic best models the perception of colors.

—Color combinations gives colors, where color components (e.g., in a RGB model) do not depend on the combined colors in a linear manner. Nonlinearity of color fusion can be best modelled by means of fuzzy concepts as fuzzy integral.

If color values are interpreted as fuzzy sets, the sets-as-points approach to fuzzy logic gives color values as points in a three-dimensional unit cube. Then, membership dominance equals the dominance relation, which was used for the definition of the Pareto set. The degree of subsethood of one color value within the other can be interpreted as "soft dominance". If two color values $C_1 = (r_1, g_1, b_1)$ and $C_2 = (r_2, g_2, b_2)$ are given (e.g., in RGB color space), then the area of the intersection is given by

$$M(C_1 \cap C_2) = \min[r_1, r_2]\min[g_1, g_2]\min[b_1, b_2].$$

The degree of dominance of C_1 over C_2 is defined by the ratio

$$\mu_D(C_1, C_2) = \frac{M(C_1 \cap C_2)}{M(C_2)} = S(C_2, C_1).$$

The degree of dominance of a over b is defined to be 0, if b is the empty fuzzy set, and 1, if a is the whole set. If C_1 dominates C_2 in the sense of Eq. (1), $C_1 \cap C_2$ equals C_2, and the ratio is 1.

For $M(C_2) = 0$, the ratio is not defined. The color value C_2 lacks at least one color component. Since $M(C_1 \cap C_2) = 0$ in this case, too, the fraction becomes a "0/0" expression, which has no definite value. There are two possibilities to assign a value: to count only subsets for the remaining dimensions; or to set it definitely to 0, since the volume of a $(n-1)$-dimensional area in the n-dimensional space is 0. For simplicity, a small positive value can be added to all color components as well, thus, avoiding the division by zero (but this seems to be a rather technical patch).

2.3.3. Fuzzy-Pareto-Morphology. So far, the strict definition of dominance has been fuzzified. Now, each pair of color values, if considered as fuzzy sets, is dominated by each other color value to a certain degree. For unifying all this information into a single ranking criterion, a fuzzy fusion operation has to be applied. We consider the most simple minmax fuzzy fusion operations of the kind

$$\arg\min[\max[\vec{f}(p)]].$$

When $\vec{f}$ is given with the set of fuzzy dominance degrees of one color value over all other color values, this leads to the definition of Fuzzy-Pareto-Morphology.

If the neigborhood M of a pixel is given by n pixels with color values x_{ij} with $i = 1, \ldots, n$ and $j = 1, 2$ or $j = 1, 2, 3$,[6] then the *Fuzzy-Pareto-Dilation* (FPD) is given by the set function:

$$\mathbf{P}_{FPD} \equiv \underset{i}{\operatorname{argmin}} \left[\max_{k \neq i} \frac{\prod_j \min(x_{ij}, x_{kj})}{\prod_j x_{ij}} \right]. \qquad (2)$$

The accompanying *Fuzzy-Pareto-Erosion* (FPE) is given as the complement of this operation according to:

$$\mathbf{P}_{FPE} \equiv \underset{i}{\operatorname{argmax}} \left[\min_{k \neq i} \frac{\prod_j \min(x_{ij}, x_{kj})}{\prod_j x_{ij}} \right]. \qquad (3)$$

Both FPD and FPE constitutes the basic operations of the *Fuzzy-Pareto-Morphology.*

Figure 3 illustrates the procedure behind the FPD at a given image location p for the two-dimensional case. The structuring element chooses the four neighbors of the pixel p and p itself. The points belonging to the color values at these five positions are marked in the unit square (Fig. 3a). These values are {(0.4, 0.6), (0.6, 0.3), (0.8, 0.1), (0.15, 0.5), (0.3, 0.15)}.

Due to property 1 (see next subsection), all dominated points are removed (Fig. 3b). These are the points (0.15, 0.3) (dominated by (0.4, 0.6)) and (0.3, 0.15) (dominated by, e.g., (0.6, 0.3)).

Then, two steps are performed. At first, to each point X of the remaining three points $A = (0.4, 0.6)$, $B = (0.6, 0.3)$, and $C = (0.8, 0.1)$ a "partner" $P(X)$ is assigned. The partner of a point X is the point $P(X)$ which is dominated by X to the highest degree. Consider point A. It is

$$S(A, B) = \frac{M(A \cap B)}{M(A)} = \frac{0.4 \cdot 0.3}{0.4 \cdot 0.6} = 0.5$$

[6] For the purpose of illustration, sometimes only two color-channels will be considered.

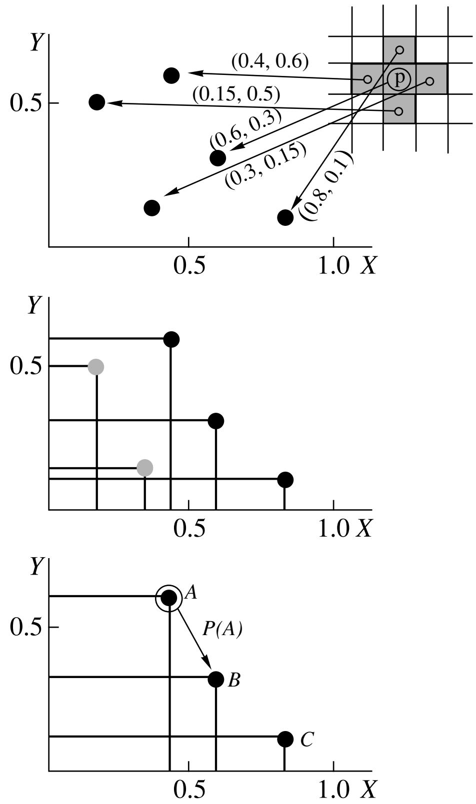


Fig. 3. Illustration of the Fuzzy-Pareto-Dilation (see text for details).

and

$$S(A, C) = \frac{M(A \cap C)}{M(A)} = \frac{0.4 \cdot 0.1}{0.4 \cdot 0.6} = 0.1\bar{6}.$$

The point A dominates B to a higher degree than C, hence, $P(A) = B$. Similarly, from $S(B, A) = 0.\bar{6}$ and $S(B, C) = 0.5$, it follows that $P(B) = A$, and from $S(C, A) = 0.5$ and $S(C, B) = 0.75$ it follows that $P(C) = B$.

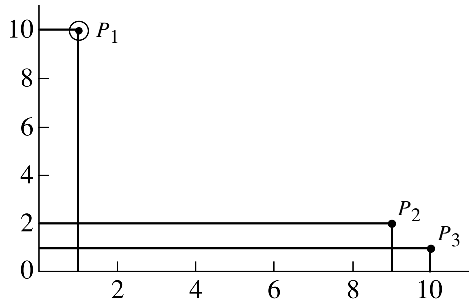
In the second step, the vector X is chosen, which gives the minimal value of $S(X, P(X))$, i.e., the one which is dominated to the lowest degree by its partner:

$$\begin{aligned} X &= \operatorname{argmin}[S(A, P(A)), S(B, P(B)), S(C, P(C))] \\ &= \operatorname{argmin}[S(A, B), S(B, A), S(C, B)] \\ &= \operatorname{argmin}[0.5, 0.\bar{6}, 0.75] = A. \end{aligned}$$

In the example, this is the point $A = (0.4, 0.6)$. The color value at position p ((0.6, 0.3)) will be replaced by the color value of its left neighbor, i.e., the position which served the point A (see Fig. 3c).

Some remarks about the definition:

There is one drawback inherited from the fuzzy subsethood definition: the degree of dominance cannot be exactly defined, when one color component is 0. See the remarks at the end of subsection 2.3.2 for the possible treatment of this case.



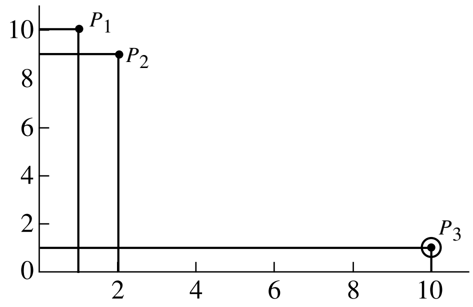


Fig. 4. Counterexample for reduced ordering.

It is useful to restrict the derivation of the terms in Eqs. (2) and (3) to differing values of p, i.e., to neglect multiple occurrences of the same color value in M.

The value in Eq. (2) depends on the chosen color model. This cannot be prevented, for there will be no ranking independent of the color model. In some cases, several color values will give the same values (e.g., a nieghborhood in an RGB-image with one point red, one point blue, and one point green). Hence, the argmin operation becomes ambiguous. There is no contradiction, because the ranking does not prefer any one of the *basic colors* (it should not do so). In this case, a totalization procedure, similar to that for the reduced ordering, can be used as well, e.g., derived from a fixed arrangement of the three basic colors. The random choose of a argmax color value would have the disadvantage that the results of the FPD could be different in areas where the neighborhoods are the same.

2.3.4. Properties of the Fuzzy-Pareto-Diation. In the following, some properties of the FPD are given.

Property 1 FPD is a Pareto-Morphology.

Proof. All dominated points go with the maximal value 1 into the competition for the lowest value of the maxima. Hopefully, a dominated point cannot win, and the winner will be an element of the Pareto set.

Property 2 The FPD is not a reduced ordering.

Proof. Consider Fig. 4. We assume, that there is a parameter function f, which gives the same ranking as the FPD ordering scheme given in Eq. (2). If we take the three points (1, 10), (9, 2), and (10, 1), the point selected by FPM is (1, 10). If we take the three points (1, 10), (2, 9), and (10, 1), the selected element will be (10, 1). But, both sets have two points in common, which exchange their ranking. If there is such an f, it must be $f(1, 10) > f(10, 1)$ from the first case, but also $f(1, 10) < f(10, 1)$ from the second case. This is not possible, hence, there is not such an f.

For a better understanding of this fact, one has to consider what is finally selected by the ordering scheme of FPM. It is the most exotic color value in the context of the other color values. The "strengths" of each color value in the competition is influenced by the presence of other color values. Two similar color values will choose each other as partners (we refer to the explained example given above) and have a high degree of mutual dominance. From this, it is improbable that one of them will give the minimum degree of being dominated by its partner. So, the alternating points (2, 9) and (9, 2) are chosen in the proof to demonstrate the weakening of the "strengths" of the color value, to which they are nearby, in the competition.

Property 3 For exactly two points, FPD is a reduced ordering.

Proof. The parameter function for ordering is the product of the components of p, as it follows from the examination of the expression in Eq. (2) for this case:

$$\operatorname*{argmin}_{\{1,2\}}\left(\frac{\prod_j \min(x_{1j}, x_{2j})}{\prod_j x_{1j}}, \frac{\prod_j \min(x_{2j}, x_{1j})}{\prod_j x_{2j}}\right).$$

However, both nominators are equal, hence, the selection is equal to

$$\operatorname*{argmax}_{\{1,2\}}\left(\prod_j x_{1j}, \prod_j x_{2j}\right),$$

i.e., the product of the components is the scalar function which gives the reduced ordering.

Now, it can be seen, how FPD is related to complete ordering (see subsection 2.2.5). Due to FPD being a reduced morphology for two points, property 3 of a complete ordering is fulfilled as well. But it must not hold that $\mathbf{P}\{x_1, x_2, x_3\} = x_1$ follows from $\mathbf{P}\{x_1, x_2\} = x_1$. Fuzzy-Pareto-Morphology is an example for a complete ordering which is not a total ordering.

Property 4 FPM reduces to grayscale morphology, if each point is a scalar value.

Proof. This follows directly from the Pareto Ordering Theorem, because the Pareto set in the one-dimensional case has just one element, the maximum value. Because FPD selects from the Pareto set, it *must* take the maximum. This is the supremum set function of grayscale dilation. In a similar manner, it can be verified that univariate FPE select the minimum value.

Property 5 FPD is not compatible with binary dilation.

Proof. Consider the structuring elements given in Fig. 5. Because each of the left structuring elements contains two points, by property 3, FPD is reduced ordering with the product of the components as scalar function. This means, that the consecutive application of both structuring elements would select the color value with the largest product of its components out of the four positions covered by the structuring element on the right side of Fig. 5. But then, the FPM would be a reduced ordering according to the product of elements for four points, too! In the proof of property 3, an example for three points was given for which this was shown to be impossible.[7]

2.3.5. Other color operations derived from fuzzy-subsethood. Based on the concept of fuzzy-subsethood, other operations for color image processing can be designed. Some examples will be given in the following:

• A color threshold operation can be defined, for which the result image is not a binary but a grayscale image. Given $p = (p_x, p_y, p_z)$, the gray value at each image position with color value v is the degree of subsethood of the color value v in p.

Despite of its simple nature, color thresholding allows for the design of useful operation, for example highly sensitive color texture filters. Consider the color textile example given in colorplate 2a. The HSI representation of this image is thresholded twice, with color threshold (0, 200, 100) and with color threshold (0, 108, 80) (the color thresholds are taken from $\{0, \ldots, g_{\max}\}$ and rescaled to [0, 1]). The result of the second thresholding is subtracted from the result of the first thresholding by pixelwise subtraction of the gray values. The resulting image is given in colorplate 2c. For comparison, the standard gray value transformation of color images is given in colorplate 2b. As can be seen from the threshold difference image 2c, the seemingly homogenuous background texture of the textile reveals its pecularities. These are due to surface faults, but also due to errors in the shading correction of the used scanning device.

• A fuzzy color image subtraction operation of two images p_1 and p_2, by which the degree of fuzzy-subsethood of the color value of p_1 in the corresponding color value in p_2, multiplied by the original color value, is assigned to each image position.

This is important for, e.g., the difinition of the morphological gradient in FPM. In gayscale morphology, the morphological gradient is the difference of dilated and eroded image by the same structuring element. However, simply subtracting two color values would introduce alien color values in the result image. By replacing subtraction with the mutual fuzzy-subsethood operation, this is prevented.[8]

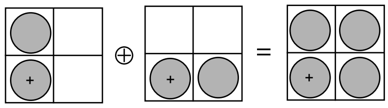


Fig. 5. Example of incompatibility of Fuzzy-Pareto-Morphology with binary morphology.

With this operation, edge operators from morphology can be used in applications.

• The selection scheme of Eq. (2) actually computes values, for which the argument leading to the smallest value is taken as a result. However, the lowest value itself can be taken as a gray value, and a grayscale image can be constructed this way (the so-called M-image).

These operations may support the processing of color images by the newly proposed FPM.

3. DISCUSSION

3.1. Comparison of Color Morphologies

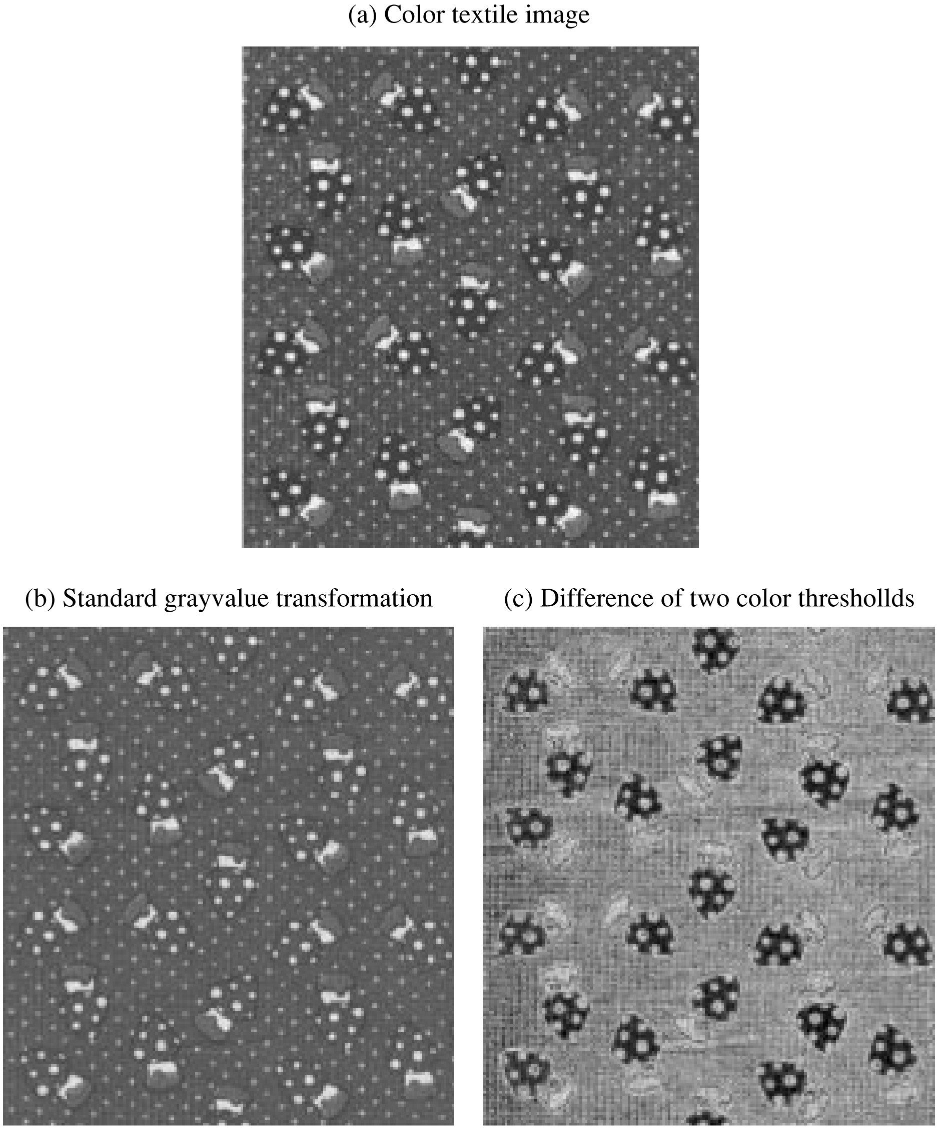
In Fig. 6, the relations between the different approaches to multivariate morphology are shown. While marginal morphology gives an isolated cluster, all other morphologies, are instances of a vector-preserving morphology. Among them, reduced morphologies, complete morphologies and Pareto-Morphologies form the basic cluster, and they partially overlap. As an example of a Pareto-Morphology, which is not a reduced morphology, the FPM is proposed in this paper.

Colorplate 1 demonstrates the effects of each class of multivariate morphologies for a synthetic test image. The test image was designed by applying the limbstone filter provided with Adobe Photoshop onto a smoothed selection within an image, which was completely filled with yellow RGB color value (200, 200, 1) (colorplate 1a). After that, blue salt and pepper noise was spread over the whole image with a unique color value (50, 50, 100) (colorplate 1b). Thus, the test image contains two regions—a two-color region of blue dots on yellow background, and a textured region, with blue dots on smoothly varying yellow color values. The different approaches to color morphology were applied to the image of colorplate 1b. This example image was provided in order to demonstrate the apparent differences in the effects of applying the different color morphologies onto images. The purpose is not a specified image processing task such as noise filtering, etc.

The result of marginal ordering is given in colorplate 1c, Clearly, the marginal ordering creates color values, which were not present within the input image. In the two-color case, the new color in the neighbor-

[7] For the fourth point, choose one which is dominated by one of our example, as (1, 9).

[8] Multiplying all components of a color value by the same scalar gives a physiologically similar appearing color.



Colorplate 2. Comparison of color thresholds and standard gray value transformation.

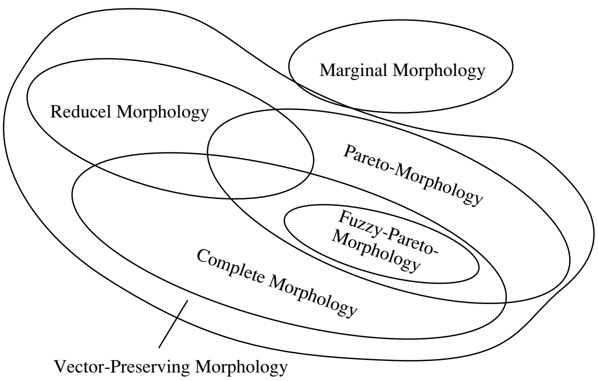


Fig. 6. Relations between different multivariate morphologies.

hood of each blue dot is (200, 200, 100). Colorplate 1d gives the result of reduced ordering with parameter function $f(r, g, b) = r + g + b$. All blue dots are filtered, because the sum of its components (200) is too small to compete with the sum of the components of the yellow color values (about 400).

The opposite effect is shown in colorplate 1e, when the parameter function for reduced ordering is $f(r, g, b) = b$, i.e., when b is preferred. Then, the blue dots are dilated, and the underlying texture becomes a little bit brighter.

Finally, colorplate 1f gives the result of the FPD. The test image was chosen so as to enhance an important property of FPM, its color context dependency. In the two-color region, the FPM is a reduced morphology with the parameter function $f(r, g, b) = rgb$. Hence, all blue dots are filtered, as in colorplate 1d. But, in the case of small variations of the yellow background, we have the situation, which is presented in Fig. 4. In the local competition among the color values, the nearly similar yellow color values mutually cancels each other. The winner is the most "exotic" color, i.e., the blue dot. In this situation, the procedure becomes very sensitive to small inhomogenities of the background. For the inner part of the texture, where the contrast is much higher, the mutual cancelation of the yellow color values is not as strong than in the bordering region of the texture, thus, the yellow color remains strong enough to win against the blue values.

The effects of relative color strengths can be seen. FPM does not assign absolute color strength to a color, as the reduced morphology does. A possible framework for making use of this property can be seen as well. The blue dots act like a probe for an image localization with a small (possibly, very small) inhomogenity in color. However, this effect depends on the actual chosen color values (here, yellow and blue). If RGB color space is used, in many cases, FPM behaves similar to a reduced morphology (see example 2 in the next subsection). This shows, that the proper choice of a color model or its transformation is an important preprocessing step for the application of color morphology in general.

3.2. Examples for Fuzzy-Pareto-Morphology

We only consider Fuzzy-Pareto-Morphology for examples, since it is the only color morphology, which is newly proposed in this parer. Examples for marginal and reduced morphologies can be found in [3, 5 and 13].

The motivation to develop a new color morphology arose from the needs of processing colored textiles. Instead of a general procedure for processing either kind of a textiled surface, it was decided to provide a toolset of operations, by the combination of which complex problems could be solved. From grayscale morphology, mathematical morphology is well known to comprise such a toolset. Hence, this versatility should be preserved for a color morphology. In the following, two examples are given for the application of the FPM in order to solve detection tasks.

Example 1 (colorplate 3a) demonstrates the detection of thread faults in a color textile. The structural property of the horizontal orientation of treads is used by the FPM. The structuring element is a vertically oriented mask of size 7. If opening, i.e., dilation followed by erosion, is applied with this mask, the thread is filtered out.

Example 2 (see colorplate 3b) demonstrates the detection of blots in a colored texture. From left to right: the original image part;[9] the result of FPM dilation with a structuring element of size 3×3; the result of applying this operation twice; and the result of applying it three times. Colorplate 3c shows the M-image of the third dilation in colorplate 3 b. The blots are clearly indicated.

[9] The arrow is just for marking the fault, but it is processed as well.

4. SUMMARY

In this paper, the Fuzzy-Pareto-Morphology (FPM) was proposed and its relation to general issues of color morphology was intensively discussed. Past theory of multivariate ranking defined four classes of multivariate ranking schemes; among them, the class of reduced ordering can be used for designing a color morphology, which is vector-preserving, i.e., it doesn't introduce new color values into the result image. Also, the concept of the Pareto set of multivariate data sets gives a means for replacing the supremum, which is used in grayscale morphology. A class of generalized morphologies, the so-called Pareto-Morphologies, was proposed, which includes generalized morphologies based on reduced ordering by a monotonic ordering function. A generalized morphology is a Pareto-Morphology if its result does not change, when its computations on the pixel nieghborhood are restricted to the Pareto set of this neighborhood of a pixel. This property extends the usually required property of a dilation to commute with the supremum.

The question, whether there is a Pareto-Morphology, which is not based on reduced ordering, got a positive answer with the proposal of the Fuzzy-Pareto-Morphology (FPM). In order to design the FPM, the concept of fuzzy subsethood of fuzzy sets within other fuzzy sets was applied to color values.

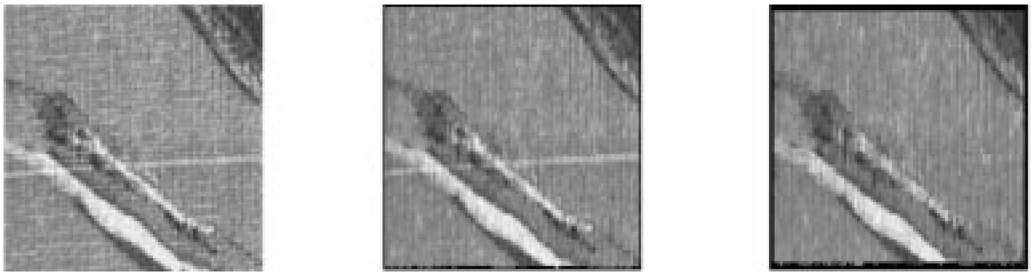
The advantage of the FPM of being not based on reduced ordering is the more natural color treatment of color images. It was shown, that FPM is vector preserving, too, and that it becomes the standard grayscale morphology, if it is considered for the case of one-dimensional data. However, the reguirement for compatibility is not fulfilled for structureless (noisy) images.

Other operations can be designed, based on the FPM (e.g., opening, closing, morphological gradient), or based on the concept of fuzzy subsethood (fuzzy subtraction of two images, color value "thresholding"). Also, intermediate results of the computations of FPM can be re-used as new image processing operators (like the M-image).

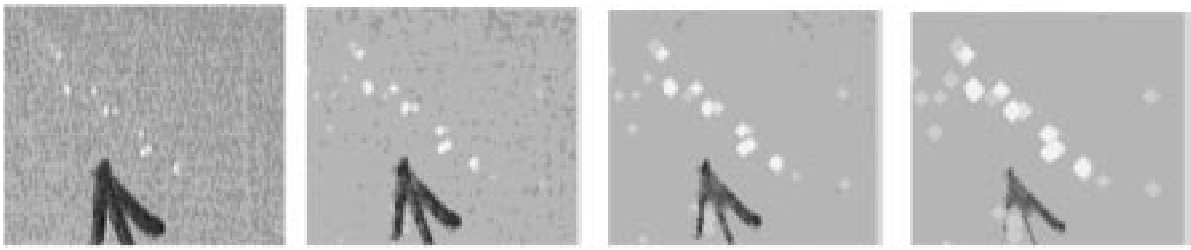
The FPM's and its accompanying operation's versatility for solving complex color image processing tasks was demonstrated by some examples, which were taken from the field of color textiles fault detection.

Currently, we are studying the interplay of different color spaces with the FPM outcome, and the adaptation of the scalar function of reduced ordering based morphologies to the color appearance in a textile image.

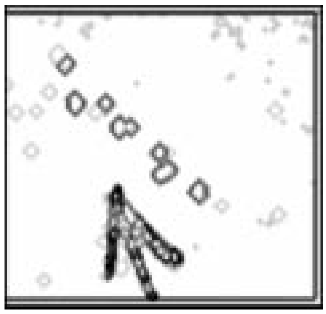
(a) Filtering of a thread dy closing with a vertical structuring element



(b) Detection of blots by repetetive application of Fuzzy-Paretto-Dilation



(c) Indication of blots by the M-image of the third dilation



Colorplate 3. Examples for the application of Fuzzy-Pareto-morphology.

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SPELL: subsethood, hul, inconvinience, toolset, thresholding, colorized, minmax, multuvariate, infimum, difinition, practive, compontent, Colorplate, explicitely, orderings, behaviour, grayscaled, powerset, monocity, iff, modelled, fuzzified, neigborhood, nieghborhood, Diation, thresholded, homogenuous, pecularities, gayscale, limbstone, inhomogenities, cancelation, inhomogenity, textiled, reguirement, enteropy, Powerest, Reducel, dy, repetetive, Paretto, Orriginal, Fuzziy, grayvalue, threshollds